Polarization and vector diffraction

Malus's law

Malus's law states that if linearly polarized light is incident upon an ideal linear polarizer, the intensity of the transmitted light is given by

$$I(\theta) = I(0)\cos^2\theta \qquad (10.79)$$

where θ is the angle between the pass-plane of the polarizer and the azimuth of the incident linear polarization and $I(0)$ is the transmitted intensity when $\theta = 0$. We can prove this result using the Jones calculus formalism developed in the previous section. Assume we have incident light that is linearly polarized in the y direction. Assuming, for simplicity, that the light has unit intensity, the incident Jones vector is

$$\mathbf{E}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (10.80)$$

The Jones matrix for a linear polarizer is given by Eq. **(2.78)**. Since the incident light is y-polarized, the angle ϕ in Eq. **(2.78)** is equivalent to the angle θ in Eq. (10.79). Using the general transformation law for the Jones calculus [Eq. **(2.74)**], we find that the Jones vector for the transmitted wave is

$$\mathbf{E}_t = \begin{bmatrix} \cos\phi\sin\phi \\ \cos^2\phi \end{bmatrix} \qquad (10.81)$$

The intensity of the transmitted light is the sum of the squared moduli of the x and y components of $\mathbf{E}_t$, i.e., $I(\phi) = |E_{tx}|^2 + |E_{ty}|^2$ or

$$\begin{aligned} I(\phi) &= \cos^2\phi\sin^2\phi + \cos^4\phi = \cos^2\phi\left(\sin^2\phi + \cos^2\phi\right) \\ &= \cos^2\phi \end{aligned} \qquad (10.82)$$

This is Malus's law, with $I(0) = 1$. We can set up a simple system in OSLO to analyze this case, using the polarization element to model the linear polarizer. We start with the pass-plane of the polarizer aligned with the incident polarization ($\phi = 0°$).

```
*LENS DATA
Malus's Law Example
 SRF      RADIUS      THICKNESS   APERTURE RADIUS       GLASS SPE   NOTE
  0        --        1.0000e+20    1.0000e+14             AIR

  1        --           --           1.000000 AS          AIR

  2        --         10.000000      1.000000 S           AIR   *

  3        --           --           1.000010 S

*POLARIZATION ELEMENT DATA
               AMPLITUDE     PHASE                AMPLITUDE     PHASE
  2      JA       --           --          JB        --           --
         JC       --           --          JD     1.000000        --

*WAVELENGTHS
CURRENT  WV1/WW1
   1    0.500000
        1.000000

*PARAXIAL SETUP OF LENS
 APERTURE
   Entrance beam radius:      1.000000    Image axial ray slope:   1.0000e-20
   Object num. aperture:    1.0000e-20    F-number:                     --
   Image num. aperture:     1.0000e-20    Working F-number:        5.0000e+19
 FIELD
   Field angle:             5.7296e-05    Object height:          -1.0000e+14
   Gaussian image height:  -1.0000e+14    Chief ray ims height:    1.0000e-05
 CONJUGATES
   Object distance:         1.0000e+20    Srf 1 to prin. pt. 1:         --
   Gaussian image dist.:   -1.0000e+20    Srf 2 to prin. pt. 2:         --
   Overall lens length:         --        Total track length:      1.0000e+20
```

```
   Paraxial magnification:    1.000000    Srf 2 to image srf:       10.000000
 OTHER DATA
   Entrance pupil radius:     1.000000    Srf 1 to entrance pup.:      --
   Exit pupil radius:         1.000000    Srf 2 to exit pupil:         --
   Lagrange invariant:     -1.0000e-06    Petzval radius:         1.0000e+40
   Effective focal length:      --

*OPERATING CONDITIONS: GENERAL
   Image surface:                    3    Aperture stop:                    1
   Evaluation mode:             Afocal    Reference surface:                1
   Aberration mode:            Angular    Aperture checking in raytrace:   On
   Number of rays in fans:          21    Designer:                      OSLO
   Units:                           mm    Program:   OSLO SIX Rev. 5.10 SIN-G
   Wavefront ref sph pos:   Exit pupil    OPD reported in wavelengths:     On
   Callback level:                   0    Print surface group data:       Off
   Compute solves in configs:      Off
   Telecentric entrance pupil:     Off    Wide-angle ray aiming mode:     Off
   Aper check all GRIN ray segs:   Off    Extended-aper ray aiming mode:  Off
   Plot ray-intercepts as H-tan U: Off    XARM beam angle:          90.000000
   Source astigmatic dist:      --        Ray aiming mode:          Aplanatic
   Temperature:              20.000000    Pressure:                  1.000000

*OPERATING CONDITIONS: POLARIZATION
   Use polarization raytrace:       On    Degree of polarization:    1.000000
   Ellipse axes ratio:          --        Y to major axis angle:      --
   Handedness of ellipse:        Right    Use 1/4 wave MgF2 coating:      Off
```

Since the incident polarization and the pass-plane of the polarizer are aligned, all of the incident light should be transmitted, as the polarization ray trace data indicates.

```
*TRACE RAY - LOCAL COORDINATES
 SRF       Y           X           Z           YANG        XANG        D
        INTENSITY  DEG. POLRZ. ELL. RATIO  ELL. ANGLE   HANDEDNESS
  1        --          --          --          --          --          --
         1.000000    1.000000      --          --          --

  2        --          --          --          --          --          --
         1.000000    1.000000      --          --          --

  3        --          --          --          --          --       10.000000
         1.000000    1.000000      --          --          --
 PUPIL     FY          FX                                              OPD
           --          --                                              --
```

We now change the pass-plane angle of the polarizer to 30° by changing the elements of the Jones matrix for surface 2.

```
*POLARIZATION ELEMENT DATA
               AMPLITUDE     PHASE              AMPLITUDE     PHASE
  2      JA     0.250000      --         JB     0.433013       --
         JC     0.433013      --         JD     0.750000       --
```

Malus's law predicts that the transmitted intensity should be $\cos^2(30°) = 0.75$. The transmitted light should be linearly polarized at an angle of 30° from the *y*-axis. Tracing a polarization ray confirms this.

```
*TRACE RAY - LOCAL COORDINATES
 SRF       Y           X           Z           YANG        XANG        D
        INTENSITY  DEG. POLRZ. ELL. RATIO  ELL. ANGLE   HANDEDNESS
  1        --          --          --          --          --          --
         1.000000    1.000000      --          --          --


  2        --          --          --          --          --          --
         0.750000    1.000000      --       30.000000      --


  3        --          --          --          --          --       10.000000
         0.750000    1.000000      --       30.000000      --

 PUPIL     FY          FX                                              OPD
           --          --                                              --
```

A pass-plane angle of 60° results in a transmitted intensity of $\cos^2(60°) = 0.25$.

*POLARIZATION ELEMENT DATA

```
               AMPLITUDE     PHASE              AMPLITUDE     PHASE
  2      JA     0.750000      --         JB     0.433013       --
         JC     0.433013      --         JD     0.250000       --

*TRACE RAY - LOCAL COORDINATES
 SRF       Y           X           Z           YANG        XANG        D
        INTENSITY  DEG. POLRZ. ELL. RATIO  ELL. ANGLE   HANDEDNESS
```

```
  1         --          --          --          --          --          --
          1.000000    1.000000      --          --          --


  2         --          --          --          --          --          --
          0.250000    1.000000      --       60.000000      --


  3         --          --          --          --          --       10.000000
          0.250000    1.000000      --       60.000000      --

 PUPIL      FY          FX                                              OPD
            --          --                                              --
```

Finally, orienting the pass-plane of the polarizer along the *x*-axis ($\phi = 90°$) results in complete attenuation of the incident light.

```
*POLARIZATION ELEMENT DATA
               AMPLITUDE     PHASE                 AMPLITUDE     PHASE
  2      JA     1.000000      --          JB        --            --
         JC       --          --          JD        --            --

*TRACE RAY - LOCAL COORDINATES
 SRF        Y           X           Z           YANG        XANG        D
         INTENSITY  DEG. POLRZ. ELL. RATIO  ELL. ANGLE   HANDEDNESS
  1         --          --          --          --          --          --
          1.000000    1.000000      --          --          --


  2         --          --          --          --          --          --
            --          --          --          --          --


  3         --          --          --          --          --       10.000000
            --          --          --          --          --

 PUPIL      FY          FX                                              OPD
            --          --                                              --
```

Fresnel rhomb

It is easy to see that the Fresnel equations [Eqs. **(2.61) - (2.64)**] predict different amounts of reflected and transmitted electric fields for *s* and *p* components, except for the case of normal incidence ($\theta_i = 0$). This means that, in general, all surfaces in an optical system act as polarization elements, to a greater or lesser degree. In many systems, this is an undesirable effect, and a great amount of effort has been extended in order to produce coatings that are insensitive to polarization. On the other hand, there are optical elements that put the differences between *s* and *p* reflection coefficients to advantageous use. One of these devices is the *Fresnel rhomb*, which is used to convert linearly polarized light to circularly polarized light.

For angles greater than the critical angle, the Fresnel reflection coefficients are complex, indicating that the reflected waves undergo a phase shift. In addition, this phase shift is different for the *s* and *p* components. It can be shown that the relative phase difference δ between *s* and *p* polarization for a totally internally reflected wave is

$$\tan\frac{\delta}{2} = \frac{\cos\theta_i\sqrt{\sin^2\theta_i - \left(\frac{n'}{n}\right)^2}}{\sin^2\theta_i} \tag{10.83}$$

where the refractive index ratio n'/n is less than 1 and the angle of incidence is greater than the critical angle, i.e., $\sin\theta_i \geq n'/n$. The maximum relative phase difference δ_m is

$$\tan\frac{\delta_m}{2} = \frac{1-\left(\frac{n'}{n}\right)^2}{2\frac{n'}{n}} \tag{10.84}$$

Fresnel demonstrated how to use the phase difference to convert linearly polarized light to circularly polarized light. The incident, linearly polarized wave is oriented such that the electric vector makes an angle of 45° with the plane of incidence. Then, the incident *s* and *p* amplitudes are equal and if the beam is totally internally reflected, the reflected amplitudes remain equal. The refractive index ratio and angle of incidence must be chosen so that the relative phase difference δ is equal to 90°. If this is to be achieved with a single reflection, Eq. (10.84) implies that (using $\delta_m/2 = 45°$, so that $\tan\delta_m/2 = 1$)

$$1 < \frac{1-\left(\frac{n'}{n}\right)^2}{2\frac{n'}{n}} \tag{10.85}$$

This means that the refractive index ratio must be

$$\frac{n'}{n} < \sqrt{2}-1 \tag{10.86}$$

or

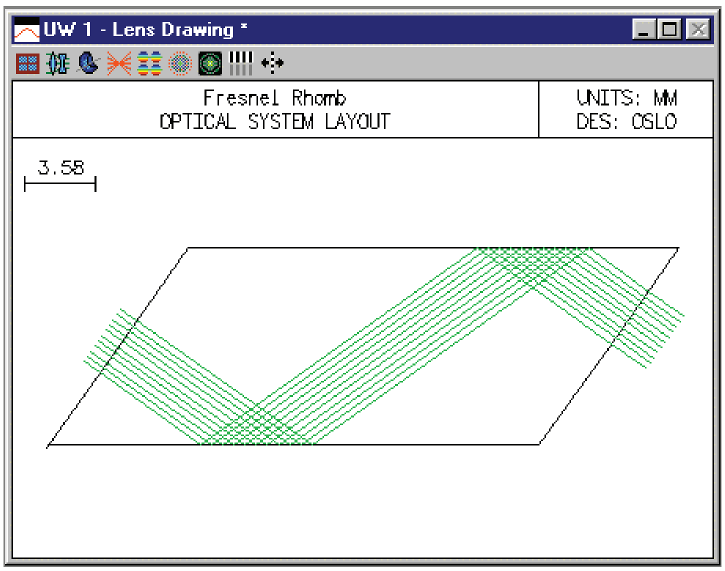
$$\frac{n}{n'} > \frac{1}{\sqrt{2}-1} = 2.4142 \tag{10.87}$$

This is not a realistic value if it is desired to use an optical glass in air. Fresnel observed that if n/n' = 1.51, then Eq. (10.84) indicates that the maximum phase difference is 45.94°, so it should be possible to choose an angle of incidence such that $\delta = 45°$ and then use two total internal reflections to achieve a total phase shift of 90°. Using n/n' = 1.51 and $\delta = 45°$ in Eq. (10.83) yields two values for the angle of incidence: $\theta_i = 48.624°$ or $\theta_i = 54.623°$. A glass block, that produces two total internal reflections at either of these two angles, is called a Fresnel rhomb.

We can enter a Fresnel rhomb in OSLO by making use of the total internal reflection only surface. We do not want to designate the surfaces as mirrors, since we need the phase shift of total internal reflection to achieve the desired change in polarization state. Using the larger value for the desired angle of incidence on the TIR surfaces, the prescription for the Fresnel rhomb is given below.

```
*LENS DATA
Fresnel Rhomb
 SRF      RADIUS      THICKNESS   APERTURE RADIUS       GLASS SPE   NOTE
  0         --       1.0000e+20    7.1006e+19            AIR

  1         --           --          2.000000 AS         AIR

  2         --       12.500000       7.912300 X        GLASS2   *
  3         --       10.000000       8.000000 X          AIR    *
  4         --       25.000000      10.000000 X          AIR    *
  5         --       10.000000       7.912300 X          AIR    *

  6         --           --         11.451742 S

*TILT/DECENTER DATA
  2     RCO   2
        DT    1        DCX        --        DCY       --       DCZ       --
                       TLA   -35.376895     TLB       --       TLC       --
  3     DT    1        DCX        --        DCY    -5.000000   DCZ       --
                       TLA    90.000000     TLB       --       TLC       --
  4     RCO   1
        DT    1        DCX        --        DCY       --       DCZ       --
                       TLA        --        TLB       --       TLC       --
  5     DT    1        DCX        --        DCY       --       DCZ       --
                       TLA   -35.376895     TLB       --       TLC       --

*SURFACE TAG DATA
  3     TIR   1
  4     TIR   1

*OPERATING CONDITIONS: GENERAL
   Image surface:                    6    Aperture stop:                     1
   Evaluation mode:             Afocal    Reference surface:                 1
   Aberration mode:            Angular    Aperture checking in raytrace:   On
   Number of rays in fans:          21    Designer:                       OSLO
   Units:                           mm    Program:   OSLO SIX Rev. 5.10 SIN-G
   Wavefront ref sph pos:  Exit pupil    OPD reported in wavelengths:     On
   Callback level:                   0    Print surface group data:       Off
   Compute solves in configs:      Off
   Telecentric entrance pupil:     Off    Wide-angle ray aiming mode:     Off
   Aper check all GRIN ray segs:   Off    Extended-aper ray aiming mode:  Off
   Plot ray-intercepts as H-tan U: Off    XARM beam angle:          90.000000
   Source astigmatic dist:      --        Ray aiming mode:          Aplanatic
   Temperature:              20.000000    Pressure:                  1.000000

*APERTURES
 SRF   TYPE APERTURE RADIUS
  0     SPC   7.1006e+19
  1     CMP     2.000000
  2     SPC     7.912300

     Special Aperture Group 0:
     A  ATP    Rectangle  AAC     Transmit  AAN       --
        AX1    -5.000000  AX2     5.000000  AY1    -6.132251  AY2     6.132251

  3     SPC     8.000000

     Special Aperture Group 0:
     A  ATP    Rectangle  AAC     Transmit  AAN       --
        AX1    -5.000000  AX2     5.000000  AY1    -8.949719  AY2    16.050281

  4     SPC    10.000000


     Special Aperture Group 0:
     A  ATP      Ellipse  AAC     Transmit  AAN       --
        AX1    -5.000000  AX2     5.000000  AY1   -16.050281  AY2     8.949719

  5     SPC     7.912300

     Special Aperture Group 0:
     A  ATP    Rectangle  AAC     Transmit  AAN       --
        AX1    -5.000000  AX2     5.000000  AY1    -6.132251  AY2     6.132251

  6     CMP    11.451742

*WAVELENGTHS
CURRENT  WV1/WW1
   1    0.500000
```

```
        1.000000

*REFRACTIVE INDICES
 SRF      GLASS            RN1         TCE
  0       AIR            1.000000       --
  1       AIR            1.000000   236.000000
  2       GLASS2         1.510000    84.000000
  3       AIR            1.000000   236.000000
  4       AIR            1.000000   236.000000
  5       AIR            1.000000   236.000000
  6       IMAGE SURFACE

*PARAXIAL SETUP OF LENS
 APERTURE
   Entrance beam radius:       2.000000    Image axial ray slope:   2.0000e-20
   Object num. aperture:     2.0000e-20    F-number:                    --
   Image num. aperture:      2.0000e-20    Working F-number:        2.5000e+19
 FIELD
   Field angle:              -35.376895    Object height:           7.1006e+19
   Gaussian image height:    7.1006e+19    Chief ray ims height:     -9.451742
 CONJUGATES
   Object distance:          1.0000e+20    Srf 1 to prin. pt. 1:        --
   Gaussian image dist.:    -1.0000e+20    Srf 5 to prin. pt. 2:        --
   Overall lens length:       47.500000    Total track length:      1.0000e+20
   Paraxial magnification:     1.000000    Srf 5 to image srf:       10.000000
 OTHER DATA
   Entrance pupil radius:      2.000000    Srf 1 to entrance pup.:      --
   Exit pupil radius:          2.000000    Srf 5 to exit pupil:      -3.311258
   Lagrange invariant:         1.420113    Petzval radius:          1.0000e+40
   Effective focal length:       --
 Note: This optical system contains special surface data.
       Calculations based on a paraxial raytrace may be invalid.
```



We select the incident polarization state by setting the polarization operating conditions to define linearly polarized light, oriented at 45° to the x and y axes.

```
*OPERATING CONDITIONS: POLARIZATION
   Use polarization raytrace:       On    Degree of polarization:     1.000000
   Ellipse axes ratio:            --      Y to major axis angle:     45.000000
   Handedness of ellipse:        Right    Use 1/4 wave MgF2 coating:       Off
Now we trace a ray from full-field (i.e., an angle of incidence of 54.623° on
surfaces 3 and 4) and observe the state of polarization as the ray passes through
the rhomb.
*TRACE REFERENCE RAY
          FBY          FBX          FBZ
        1.000000        --           --
         FYRF         FXRF          FY           FX
          --           --           --           --
          YCA          XCA          YFSA         XFSA         OPL
          --           --      -8.1536e-18  -8.1536e-18    58.264253

*TRACE RAY - LOCAL COORDINATES
 SRF          Y/L          X/K          Z/M        YANG/IANG    XANG/RANG      D/OPL
          INTENSITY   DEG. POLRZ.  ELL. RATIO   ELL. ANGLE   HANDEDNESS
  1           --           --           --       -35.376895      --           --
```

```
         -0.578952       --        0.815361   35.376895   35.376895       --
          1.000000    1.000000       --       45.000000       --

  2          --          --          --          --           --          --
             --          --        1.000000      --           --          --
          0.958715    1.000000       --       45.000000       --
  3       5.458304       --       8.8818e-16  -54.623105      --        8.636288
         -0.815361       --        0.578952   54.623105   54.623105   13.040795
          0.958715    1.000000     0.414214   44.999999     LEFT =    -1.000000
  4      -8.625087       --          --      -125.376895      --       17.272577
         -0.815361       --       -0.578952  -54.623105  -54.623105   39.122386
          0.958715    1.000000     1.000000  -27.956108     LEFT =    -1.000000
  5       1.833416       --          --          --           --        6.054216
             --          --        1.000000      --           --       48.264253
          0.919134    1.000000     1.000000  -27.956110     LEFT =    -1.000000

  6       1.833416       --          --          --           --       10.000000
             --          --        1.000000      --           --       58.264253
          0.919134    1.000000     1.000000  -27.956110     LEFT =    -1.000000
 PUPIL       FY          FX                                               OPD
             --          --                                           1.4211e-11
```

After the first total internal reflection (surface 3), the light is elliptically polarized; the ratio of the axes of the polarization ellipse is 0.414. After the second reflection (surface 4), the light is circularly polarized; the ratio of the axes is 1.0. The wave is normally incident on the end faces of the rhomb (surfaces 2 and 5), so the polarization state is not changed at these surfaces, but some of the incident intensity is lost due to reflection losses of about 4% at each surface.

The Fresnel rhomb can also be used “in reverse.” If circularly polarized light is incident upon the rhomb, linearly polarized light exits the rhomb.

```
*OPERATING CONDITIONS: POLARIZATION
   Use polarization raytrace:       On    Degree of polarization:    1.000000
   Ellipse axes ratio:        1.000000    Y to major axis angle:       --
   Handedness of ellipse:         Left    Use 1/4 wave MgF2 coating:      Off

*TRACE REFERENCE RAY
        FBY         FBX          FBZ
      1.000000      --           --
       FYRF        FXRF          FY          FX
        --          --           --          --
        YCA         XCA         YFSA        XFSA        OPL
        --          --     -8.1536e-18 -8.1536e-18   58.264253

*TRACE RAY - LOCAL COORDINATES
 SRF       Y/L         X/K         Z/M      YANG/IANG   XANG/RANG      D/OPL
         INTENSITY  DEG. POLRZ. ELL. RATIO  ELL. ANGLE  HANDEDNESS
  1          --          --          --      -35.376895      --          --
         -0.578952       --        0.815361   35.376895   35.376895       --
          1.000000    1.000000     1.000000    3.925177     LEFT =    -1.000000

  2          --          --          --          --           --          --
             --          --        1.000000      --           --          --
          0.958715    1.000000     1.000000    3.925177     LEFT =    -1.000000
  3       5.458304       --       8.8818e-16  -54.623105      --        8.636288
         -0.815361       --        0.578952   54.623105   54.623105   13.040795
          0.958715    1.000000     0.414214  -45.000001     LEFT =    -1.000000
  4      -8.625087       --          --      -125.376895      --       17.272577
         -0.815361       --       -0.578952  -54.623105  -54.623105   39.122386
          0.958715    1.000000    4.9216e-16  -44.999999    RIGHT =     1.000000
  5       1.833416       --          --          --           --        6.054216
             --          --        1.000000      --           --       48.264253
          0.919134    1.000000    4.9216e-16  -45.000001    RIGHT =     1.000000

  6       1.833416       --          --          --           --       10.000000
             --          --        1.000000      --           --       58.264253
          0.919134    1.000000    4.9216e-16  -45.000001    RIGHT =     1.000000
 PUPIL       FY          FX                                               OPD
             --          --                                           1.4211e-11
```

Wollaston Prism

OSLO Premium Edition has the capability of tracing rays through uniaxial birefringent materials such as calcite. Two types of waves (and rays) can propagate in uniaxial media; these waves are called the *ordinary* wave (or *o*-ray) and the *extraordinary* wave (or *e*-ray). Ordinary waves and rays can be traced using the same techniques as are used for ray tracing in isotropic media. Tracing extraordinary rays, however, is more complicated. The refractive index for extraordinary rays is a function of the angle of incidence. Also, for the extraordinary ray, the wave vector (the normal vector to the wavefront) is generally not in the same direction as the ray vector (the vector in the direction of energy flow, i.e., the Poynting vector).

The interaction of an electric field with a material is characterized by the permittivity (dielectric constant) ε of the material. The permittivity relates the electric field **E** to the electric displacement **D**. For the nonmagnetic materials used in optical systems, *Maxwell's relation* states that the refractive index n is equal to the square root of the permittivity, i.e., $n^2 = \varepsilon$. For isotropic materials, the permittivity is a scalar quantity (although a function of wavelength). By contrast, the permittivity of an anisotropic medium such as a crystal must be described by a tensor. In other words, $\boldsymbol{\varepsilon}$ is a 3 x 3 matrix that relates the components of **E** to the components of **D**. (The difference between the wave vector and the ray vector for the extraordinary ray is a consequence of **D** no longer being collinear with **E**.)

Since the refractive index is no longer a constant at a particular point in the material, the medium is termed *birefringent*, since the index of refraction depends on the propagation direction. For the crystal materials under consideration here, a coordinate system can be found in which only the diagonal elements of the *dielectric tensor* are non-zero. The coordinate axes of this system are called the *principal axes* and the diagonal elements of the tensor are called the *principal values* of the permittivity. For *uniaxial* media, two of the principal values are equal. For *biaxial* media, all three of the principal values are different. (Note that OSLO does not treat biaxial materials.) For a uniaxial material, the axis along which the permittivity differs is the crystal axis, i.e., the axis of symmetry of the crystal. The principal values and principal axes define the *index ellipsoid*. In order to trace rays through this uniaxial birefringent medium, then, we must specify the ordinary refractive index, the extraordinary refractive index, and the orientation of the crystal axis.

In OSLO, the data for the ordinary indices is taken from the normal glass specification for the surface. To specify that a medium is birefringent, click the glass options button for the desired surface, and select Birefringent medium from the pop-up menu. In this spreadsheet, you can specify the material that defines the extraordinary refractive indices and the orientation of the crystal axis.

The extraordinary indices may either be calculated from a catalog material, or the indices may be specified explicitly. Click the appropriate radio button to specify your choice. The orientation of the crystal axis is determined by the specification of direction numbers for the axis. The direction numbers (denoted by CAK, CAL, and CAM, which are the direction numbers in x, y, and z, respectively) are the Cartesian components of a vector in the direction of the crystal axis. If the direction numbers are normalized (i.e., the magnitude of the vector is unity), the direction numbers are the direction cosines of the crystal axis. It is not necessary, however, to enter the direction numbers in normalized form. For example, if the crystal axis is parallel to the y-axis of the surface, the direction numbers would be CAK = 0, CAL = 1, CAM = 0. (For this case, where CAK = CAM =0, CAL could be any non-zero value.) For birefringent materials, the crystal axis direction numbers may be made optimization variables and operand components.

In general, a ray incident upon a birefringent material will be split into two rays (*o* and *e*), with orthogonal linear polarizations. Since OSLO does not split rays, you must specify which ray is to be traced through the material. This designation is also made in the birefringent medium spreadsheet. By default, the ordinary ray is traced. The easiest way to see the results of tracing the other ray is to make the system a multiconfiguration system, where the configuration item is the ray that is traced through the medium (the name of the configuration item is BRY).

As mentioned above, for the extraordinary ray the wave vector and the ray vector are generally not in the same direction. Thus, we need two sets of direction cosines to characterize the propagation of the ray through the medium. In OSLO, the direction cosines (K, L, and M) reported in the trace_ray command (Evaluate >> Single Ray) correspond to the *ray* vector. Similarly, the RVK, RVL, and RVM operands refer to the data for the ray vector. If an extraordinary ray is being traced, the output of the trace_ray command will contain three more columns of numbers (columns 7, 8, and 9 of the spreadsheet buffer). These columns contain the values of the direction cosines for the *wave* vector. These columns are labeled LWV, KWV, and MWV, keeping with the OSLO convention of outputting the *y* value before the *x* value. If it is desired to use the wave vector direction cosines in ray operands, the components KWV, LWV, and MWV are available. For ordinary rays in birefringent media and for rays in isotropic media, the KWV, LWV, and MWV components have the same value as the RVK, RVL, and RVM components.

Five common uniaxial materials are contained in the miscellaneous glass catalog: calcite ($CaCO_3$), ADP, KDP, MgF_2, and sapphire (Al_2O_3). With the exception of the *o*-indices for calcite, the dispersion equations for these materials were taken from the *Handbook of Optics*, Volume II, Chapter 33, "Properties of Crystals and Glasses", by W. J. Tropf, M. E. Thomas, and T. J. Harris, Table 22 (McGraw-Hill, New York, 1995). The calcite *o*-index dispersion equation data was calculated by performing a least-squares Sellmeier fit to the data in Table 24 of the reference. The RMS error of this fit is 0.000232, as compared to the tabulated values, over the wavelength range from 0.200 µm to 2.172 µm represented by the values in Table 24. (Also, several minor typographical errors in Table 22 have been corrected. The equations for calcite should be in terms of n^2 not n. The absorption wavelength in the third term for the *e*-index of MgF_2 should be 23.771995, not 12.771995.) Note that each material corresponds to two entries in the glass catalog: one for the *o*-indices (CALCITE_O, ADP_O, KDP_O, MGF2_O, SAPPHIRE_O) and one for the *e*-indices (CALCITE_E, ADP_E, KDP_E, MGF2_E, SAPPHIRE_E).

As an example of the use of birefringent materials, consider the following system, which is a Wollaston prism. This prism consists of two wedges of a birefringent material, in this case calcite. In the first wedge, the crystal axis is in the *y*-direction, while in the second wedge, the crystal axis is in the *x*-direction. With this orientation of crystal axes, an *o*-ray in the first wedge becomes an *e*-ray in the second wedge, and vice-versa. If a beam of circularly polarized light is normally incident on the prism, two beams exit the prism: one deflected upwards, with horizontal linear polarization, and the other downwards, with vertical linear polarization.

```
*LENS DATA
Wollaston Prism
 SRF      RADIUS      THICKNESS   APERTURE RADIUS       GLASS  SPE  NOTE
 OBJ       --       1.0000e+20    1.0000e+14           AIR

 AST       --           --         10.000000 A         AIR

  2        --         5.773503     10.000000       CALCITE_O CB
  3        --         5.773503     11.547005       CALCITE_O CB*
  4        --         3.000000     10.000000           AIR

 IMS       --           --          8.000010 S

*TILT/DECENTER DATA
  3     RCO   3
        DT    1        DCX        --        DCY        --        DCZ        --
                       TLA   -30.000000    TLB        --        TLC        --

*SURFACE TAG DATA
  3     DRW ON


*WAVELENGTHS
CURRENT  WV1/WW1
   1    0.589290
        1.000000

*REFRACTIVE INDICES
 SRF     GLASS           RN1        TCE
  0      AIR          1.000000      --
  1      AIR          1.000000  236.000000
  2      CALCITE_O    1.658342      --
 EXTRA   CALCITE_E    1.486440
 BRY       Ordinary   Crystal Axis (K,L,M)      --        1.000000      --
  3      CALCITE_O    1.658342      --
 EXTRA   CALCITE_E    1.486440
 BRY  Extraordinary   Crystal Axis (K,L,M)    1.000000      --          --
```

```
 4       AIR              1.000000   236.000000
 5       IMAGE SURFACE

*CONFIGURATION DATA
TYPE   SN    CFG   QUALF        VALUE
BRY     2     2      0            EXT
BRY     3     2      0            ORD
```